

# AmeriMech Symposium on Fluid Transport and Nonlinear Dynamics

**Date:** May 17-20, 2016 (Tue-Fri)

**Venue:** Woods Hole Oceanographic Institution, Clark Bldg, Woods Hole, Massachusetts, USA.

**Organizers:** George Haller, ETH Zürich, [georgehaller@ethz.ch](mailto:georgehaller@ethz.ch);  
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## Overview

From	To	Tuesday	Wednesday	Thursday
9:00 AM	10:30 AM	Session 1	Session 3	Session 6
10:30 AM	11:00 AM	Coffee	Coffee	Coffee
11:00 AM	12:30 PM	Session 2	Session 4	Session 7
12:30 PM	2:00 PM	Lunch break	Lunch break	Lunch break
2:00 PM	3:30 PM	Poster Session	Session 5	Open Discussions: Computations
3:30 PM	4:00 PM	Coffee	Coffee	Coffee
4:00 PM	5:30 PM	Poster Session	Open Discussions: Observations	
6:30 PM			Conference Dinner	

From	To	Friday
9:00 AM	10:00 AM	Session 8
10:00 AM	10:30 AM	Coffee
10:30 AM	12:00 PM	Session 9

# Session Overviews

## Session 1

*Tuesday, 17 May 2016, 9:00 AM to 10:30 AM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
9:00 AM	9:30 AM	Amy Bower	Woods Hole Oceanographic Institution	A Review of High-Resolution Subsurface Lagrangian Observations in the Ocean
9:30 AM	10:00 AM	Ivan Savelyev	US Naval Research Lab	Characterizing upper ocean hydrodynamics using infrared and hyperspectral airborne remote sensing
10:00 AM	10:30 AM	James Ledwell	Woods Hole Oceanographic Institution	Stirring of Tracers within the Stratified Ocean

## Session 2

*Tuesday, 17 May 2016, 11:00 AM to 12:30 PM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
11:00 AM	11:30 AM	Tom Solomon	Bucknell University	Coherent structures in reactive front propagation and bacterial swimming in fluid flows
11:30 AM	12:00 PM	Amala Mahadevan	Woods Hole Oceanographic Institution	Coherent pathways for vertical transport of carbon and oxygen from the ocean surface to depth
12:00 PM	12:30 PM	Larry Pratt	Woods Hole Oceanographic Institution	Computing and Visualizing Residual Property Fluxes

## Session 3

*Wednesday, 18 May 2016, 9:00 AM to 10:30 AM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
9:00 AM	9:30 AM	Igor Kamenkovich	RSMAS - University of Miami	Properties and Origins of the Anisotropic Eddy-Induced Transport in the Oceans
9:30 AM	10:00 AM	Anthony Kirincich	Woods Hole Oceanographic Institution	The Effects of Submesoscale Eddies on Exchange Across the Inner Shelf.

10:00 AM	10:30 AM	Irina Rypina	Woods Hole Oceanographic Institution	Investigating the eddy diffusivity concept in the coastal ocean
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## Session 4

*Wednesday, 18 May 2016, 11:00 AM to 12:00 PM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
11:00 AM	11:30 AM	John Dabiri	Stanford University	Coherent Structure Identification from Sparse Flow Trajectories using Graph Theory
11:30 AM	12:00 PM	Miles Sundermeyer	University of Massachusetts Dartmouth	Observations of Dye Dispersion in the Gulf Stream Core and Across the North Wall

## Session 5

*Wednesday, 18 May 2016, 2:30 PM to 3:30 PM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
2:30 PM	3:00 PM	Francisco Beron-Vera	RSMAS, University of Miami	Nonlocal pair dispersion in the Gulf of Mexico
3:00 PM	3:30 PM	Joe LaCasce	University of Oslo	Deducing energy spectra from drifters

## Session 6

*Thursday, 19 May 2016, 9:00 AM to 10:30 AM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
9:00 AM	9:30 AM	Michel Speetjens	Eindhoven University of Technology	Lagrangian transport in three-dimensional unsteady flows with (perturbed) periodic lines
9:30 AM	10:00 AM	Wenbo Tang	Arizona State University	Scalar density evolution with Lagrangian measures
10:00 AM	10:30 AM	Daniel Karrasch	Technische Universität München	A spectral clustering approach to coherent Lagrangian vortex detection

## Session 7

*Thursday, 19 May 2016, 11:00 AM to 12:30 PM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
11:00 AM	11:30 AM	Mohammad Farazmand	Massachusetts Institute of Technology	Polar rotation angle identifies vortices in unsteady flows
11:30 AM	12:00 PM	George Haller	ETH Zürich	Uncovering Lagrangian vortices objectively from the vorticity
12:00 PM	12:30 PM	Melissa Green	Syracuse University	Objective detection of vortices and their evolution in 2D and 3D flows

## Session 8

*Friday, 20 May 2016, 9:00 AM to 10:00 AM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
9:00 AM	9:30 AM	Ryan Abernathey	Columbia University	Lagrangian Coherent Structures and Eulerian Eddy Fluxes in the East Pacific
9:30 AM	10:00 AM	A. D. Kirwan, Jr.	SMSP, University of Delaware	Dynamics of time-dependent transport boundaries in rotating stratified Euler flow

## Session 9

*Friday, 20 May 2016, 10:30 AM to 12:00 PM*

*20 minutes per talk, 10 minutes for questions and speaker change*

From	To	Name	Affiliation	Title
10:30 AM	11:00 AM	Themis Sapsis	Massachusetts Institute of Technology	A minimization principle for the description of modes associated with finite-time instabilities
11:00 AM	11:30 AM	Sanjeeva Balasuriya	University of Adelaide	Transport between two fluids across their mutual flow interface: the streakline approach
11:30 AM	12:00 PM	Nicholas Ouellette	Stanford University	Hyperbolic Neighborhoods in Unsteady Flow

# Presentation Abstracts

## Session 1 Abstracts

### **A Review of High-Resolution Subsurface Lagrangian Observations in the Ocean**

*Amy Bower (Woods Hole Oceanographic Institution)*

For more than four decades, physical oceanographers have been tracking fluid particle motion using acoustically tracked, neutrally buoyant floats. The existence of a mid-depth minimum in sound speed—the sound or SOFAR (Sound Fixing And Ranging) channel—allows for tracking over large spatial scales (>1000 km) with high temporal/spatial resolution (< 1 day/< 5 km). The original SOFAR float, developed in the early 1970s, was replaced by the lighter, smaller, more manageable, cost-effective RAFOS float system in the late 1980s, with several variations emerging in the following decades. Hundreds of trajectories have been collected from high latitudes to the tropics, at depths from near-surface to the abyss, in the Atlantic, Pacific, Indian and Southern Oceans. Depending on the research topic, trajectories vary in length from months to years, and can track 2D or 3D fluid motion.

This talk will review some of the unique discoveries made using these Lagrangian observing tools, summarize the technology and plans for future development, and describe new results from a study of the deep circulation in the Gulf of Mexico. The high-resolution (compared to typical Lagrangian integral time scales) trajectories have been particularly enlightening in studies of (a) the formation, propagation, kinematics and dynamics of mesoscale and submesoscale subsurface eddies; (b) dispersion and Lagrangian statistics; (c) quasi-Eulerian statistics over large spatial regions; and (d) the structure of narrow boundary currents and water mass spreading pathways. Information on all four of these areas is being gleaned from a set of 180 RAFOS floats deployed in the Gulf of Mexico in the depth range 1500-2500 m during 2011-2015.

### **Characterizing upper ocean hydrodynamics using infrared and hyperspectral airborne remote sensing**

*Ivan Savelyev (US Naval Research Lab)*

This presentation will describe a series of airborne remote sensing experiments conducted by the Naval Research Lab (NRL) over the coastal ocean. While NRL conducts airborne tests using widest variety of active and passive sensors across the EM spectrum, for the coastal applications the most practical sensors are often found to be mid-wave infrared and hyperspectral visible-near IR passive sensors, which will be covered in more detail here. Recent improvements in infrared and hyperspectral imaging technologies, as well as in high grade positioning systems, enabled airborne remote sensing to become a relatively inexpensive and highly effective component in coastal ocean studies. Capturing scales from meters to 1000s of meters, airborne remote sensing has been shown to resolve spatial, as well as temporal structures of such processes as submesoscale eddies, fronts and frontal instabilities, wakes behind islands and submerged mounts, breaking surface and internal waves, rip currents, convection and Langmuir cells, among others. Spatially rich data provided by an airborne sensor is often unique and unattainable by any other conventional oceanographic instrument. On the other hand, it is often hard to

quantify airborne imagery in terms of commonly used oceanographic parameters, making concurrent in-water ground truth measurements an essential counterpart of an airborne survey.

### **Stirring of Tracers within the Stratified Ocean**

*James Ledwell (Woods Hole Oceanographic Institution)*

Glimpses of the dispersion of tracer in the stratified ocean interior have been obtained from a number of tracer release experiments. As in most other fluid stirring situations, streaks of tracer develop, indicating the action of high strain and in the direction orthogonal to streaks barriers to transport are apparent. The processes seem to prevail at the full range of scales investigated, from the 100-m scale of the initial releases, to the 100-km scale of mesoscale eddies. Rates of straining at these various scales can be crudely estimated from the evolution of the tracer fields, as can rates of dispersion across the barriers. Results from experiments in the main pycnocline of the North Atlantic, the salt finger staircase in the main pycnocline east of Barbados, the deep Brazil Basin, the mid-depth Gulf of Mexico, and the Circumpolar Deep Water of the Antarctic Circumpolar Current will be presented. In the case of the Antarctic Circumpolar Current, inhibition of transport of tracer across the Polar Front will be discussed.

## Session 2 Abstracts

### **Coherent structures in reactive front propagation and bacterial swimming in fluid flows**

*Tom Solomon (Bucknell University)*

We present experiments on the behavior of propagating reaction fronts and swimming bacteria in laminar fluid flows. Two- and three-dimensional magnetohydrodynamically-driven flows are used for the reaction experiments, and a microfluidic flow in a T-channel is used for the bacteria studies. Reaction fronts are produced by the excitable Belousov-Zhabotinsky chemical reaction, and the bacteria studied are a smooth-swimming mutation of *Bacillus subtilis*. All of these experiments are analyzed in terms of burning invariant manifolds (BIMs) and swimming invariant manifolds (SWiMs) that act as local, one-way barriers that impede the motion of the reaction front or of the swimming bacteria. We have completed several studies showing the importance of BIMs for understanding reaction propagation in 2D flows. More recently, we have extended these ideas to 3D flows in which the BIMs take the form of tubes or sheets, which are also one-way in their blocking behavior. The bacteria experiments have only recently been initiated, but preliminary experiments have shown evidence of one-way barriers consistent with predictions of SWiMs.

### **Coherent pathways for vertical transport of carbon and oxygen from the ocean surface to depth**

*Amala Mahadevan (Woods Hole Oceanographic Institution)*

Transport within the oceanic eddy field is largely horizontal; vertical velocities are several orders of magnitude smaller than horizontal velocities at scales of 1-100 km. But, the vertical transport of water and biogeochemical properties is crucial for the oceanic biological pump. Here, I describe how water

from the surface mixed layer of the ocean is conveyed to depth in coherent features formed along the edges of eddies. Using glider observations of oxygen and particulate organic carbon from the North Atlantic Bloom experiment, we identify tongue-shaped filaments of carbon- and oxygen-rich waters that are subducted from the surface. Process modeling experiments show that eddies and fronts provide downwelling pathways along sloping isopycnal surfaces. We derive a scaling relationship to quantify the eddy-driven subduction rate of phytoplankton carbon from the oceanic mixed layer, and infer that this mechanism contributes to significant export of carbon and oxygen from the surface ocean to depth.

### **Computing and Visualizing Residual Property Fluxes**

*Larry Pratt (Woods Hole Oceanographic Institution)*

Dynamical systems analysis provides for the mapping and visualization of fluid transport pathways and barriers. Some techniques such as lobe analysis or elliptical boundary identification can lead to quantification of volume transport between different regions of a flow field. Although a knowledge of volume transport may be valuable, one is often more interested in the fluxes and budgets of scalar properties such as heat, vorticity or nutrients. In addition, the presence of small-scale structures in the flow field, particularly when sub-mesoscales are resolved, may render techniques such as lobe analysis impractical. I will describe a new approach designed to address these difficulties.

## **Session 3 Abstracts**

### **Properties and Origins of the Anisotropic Eddy-Induced Transport in the Oceans**

*Igor Kamenkovich (RSMAS - University of Miami)*

This study examines properties and origins of the eddy-induced transport in oceanic flows, using idealized models of the double-gyre oceanic circulation and altimetry-derived velocities. The transport due to the time-dependent flow (“eddies”) is quantified by a two-dimensional, location-dependent diffusivity tensor, taking into account modulations by the stationary (“mean”) flow. This transport is highly anisotropic, that is, it has a well-defined direction of the maximum transport. The anisotropy is only partly explained by the effects of the mean advection, and is in large part due to the eddying field. One component of the time-dependent flow, zonally-elongated large-scale transients (ZELTs), is particularly important for the anisotropy, as it corresponds to primarily zonal material transport and long correlation time scales. The mechanism is analogous to shear dispersion, as is further illustrated by idealized simulations of an eddying zonal flow. The importance of ZELTs is further confirmed by simulations of idealized color dye tracers, which has important implications for parameterizations of the eddy-induced transport in non-eddy-resolving models.

## **The Effects of Submesoscale Eddies on Exchange Across the Inner Shelf**

*Anthony Kirincich (Woods Hole Oceanographic Institution)*

This work investigates the characteristics and implications of sub-mesoscale variability over the continental shelf in order to understand the relative importance of lateral mechanisms of exchange and stirring on the total across-shelf transport. Within the study area south of Martha's Vineyard, USA, observations of high-resolution HF radar surface currents allow lateral scales as small as 1.5 km to be resolved within a 30x30 km domain. Coupled with dense observations of subsurface velocity and hydrography for a 6-month period spanning both stratified and weakly stratified conditions, these observations were used to document horizontal and vertical scales, occurrences, and drivers of spatially-variable circulation. Coherent vortices, or eddies, driven both by density intrusions and tidal processes were observed at rates up to 4 per day during the stratified period, with locations and length scales dependent on wind forcing. Despite being temporally short, with mean durations of 5 hours, based on volumetric transport rates, these features caused exchange equivalent to the wind-driven depth-dependent exchange.

## **Investigating the eddy diffusivity concept in the coastal ocean**

*Irina Rypina (Woods Hole Oceanographic Institution)*

We test the validity, utility, and limitations of the lateral eddy diffusivity concept in a coastal environment through analyzing data from coupled drifter and dye releases within the footprint of a high-resolution (800 m) high-frequency radar south of Martha's Vineyard MA. Specifically, we investigate how well a combination of radar-based velocities and drifter-derived diffusivities can reproduce observed dye spreading over an 8-hour time interval. A drifter-based estimate of an anisotropic diffusivity tensor is used to parameterize small-scale motions that are un- and under-resolved by the radar system. This leads to a significant improvement in the ability of the radar to reproduce the observed dye spreading.

## **Session 4 Abstracts**

### **Coherent Structure Identification from Sparse Flow Trajectories using Graph Theory**

*John Dabiri (Stanford University)*

Experimental fluid mechanics is trending toward widespread use of three-component, three-dimensional flow velocimetry, which is typically based on measurement of Lagrangian flow trajectories. This type of data can present challenges for standard methods of coherent structure identification that rely on measurement of the deformation gradient, as the requirement for initially closely-spaced trajectories is often not satisfied. Moreover, the relatively long duration over which fluid trajectories are followed may call into question the physical relevance of linearized flow maps. In this work, we develop a technique for coherent structure identification that does not require assumptions of initially closely-spaced trajectories or a linearized flow map. Moreover, the method remains effective for very small



numbers of flow trajectories relative to the requirements of other Lagrangian techniques. Example applications of the method will be presented along with opportunities to further improve its effectiveness.

### **A coherent structure approach for parameter estimation in Lagrangian Data Assimilation**

*John Maclean (University of North Carolina at Chapel Hill)*

The motivation for this work comes from oceanographic problems amenable to data assimilation, in which the observations are of passive tracers. We consider a formulation of data assimilation in which the forecasts and observations are processed to identify spatial coherent patterns within the tracer trajectories. These patterns may consist of, for instance, large scale eddies. We employ an Approximate Bayesian Computation to compare the patterns, and suggest that this is an efficient way to perform accurate parameter estimation in systems with many observations of the tracers, provided that spatial coherent patterns do exist. The key assumption is that the number of tracers is large enough that one can distinguish between similar spatial patterns in a local range of parameter values.

### **Observations of Dye Dispersion in the Gulf Stream Core and Across the North Wall**

*Miles Sundermeyer (University of Massachusetts Dartmouth)*

We report on a series of dye and drifter releases performed in the Gulf Stream core and along its north wall in Winter 2012, two near the surface (~26-28 m) and two at depth (~55 m, and ~120 m). The primary goal was to quantify and identify processes controlling submesoscale lateral dispersion across a strong front, including during strong forcing conditions. Dye injections were performed in concert with Lagrangian float deployments, each of which were tracked for between 1-5 days. Sampling focused on hydrography, velocity, and dye distributions in a moving reference frame around the Lagrangian float, with repeat transects through the dye patch and across the stream. Results reveal mechanisms and pathways of transport and mixing across a strong front, including evidence of the maintenance of the front in spite of such mixing. Processes observed include rapid subduction along isopycnals due to symmetric instability, and the formation and detachment of streamers and/or filaments along the north wall. To the extent possible given incomplete surveys of the dye patches, we estimate bounds on along- and cross-isopycnal dispersion on scales comparable to the dye patch, i.e., hundreds of meters to 10s of km, accounting for the fact that isopycnals varied from nearly vertical (outcropping) to nearly horizontal (main pycnocline).

## Session 5 Abstracts

### **Cluster-based, finite-time partitions for identifying Lagrangian transport properties**

*Andrew Poje (City University of New York, CSI)*

We apply a cluster-based approach to identify the temporal evolution of Lagrangian structures in a classic oceanographic transport problem, namely the cross-stream flux induced by the interaction of a meso-scale Gulf Stream Ring eddy with the main jet. The model for the highly time-dependent, motion of the Gulf Stream is a nominally 2 km, submesoscale permitting HYCOM simulation of the North Atlantic. The focus is on a single mixing event driven by the interaction between an energetic cold core ring (a cyclone), the strong jet, and a number of smaller scale cyclones and anticyclones. The timescales of interest are 1-3 weeks corresponding to model lifetimes of the larger eddies. The goal is to characterize and organize the time-dependent Lagrangian kinematics in both the horizontal and vertical directions. The approach is based on computing time-averages of fixed spatial basis functions over the trajectories and then applying standard clustering algorithms to identify the finite-time structures within which trajectories possess similar statistics. The resulting partitions are found to be robust to changes in the specifics of both the basis sets and the clustering parameters. In addition to the synthetic model trajectories, applications of the clustering method to a set of 300 in-situ surface trajectories from the Lagrangian Submesoscale Experiment, recently conducted in the northern Gulf of Mexico, will be discussed.

### **Nonlocal pair dispersion in the Gulf of Mexico**

*Francisco Beron-Vera (RSMAS, University of Miami)*

Pair-separation statistics of in-situ and synthetic surface drifters deployed near the Deepwater Horizon site in the Gulf of Mexico are investigated. The synthetic trajectories derive from a submesoscale-permitting Navy Coastal Ocean Model (NCOM) simulation. The in-situ drifters were launched in the Grand Lagrangian Deployment (GLAD). Various measures of the dispersion are calculated and compared to theoretical predictions. For the NCOM pairs, the measures indicate nonlocal pair dispersion at the smallest sampled scales. At separations exceeding 100 km, pair motion is uncorrelated, indicating absolute rather than relative dispersion. With the GLAD drifters however the statistics suggest local dispersion (in which pair separations exhibit power law growth), in line with previous findings. The disagreement stems in part from inertial oscillations, which affect the energy levels at small scales without greatly altering the net particle displacements. They were significant in GLAD but much weaker in the NCOM simulation. In addition the GLAD drifters were launched close together, producing few independent realizations and hence weaker statistical significance. Restricting the NCOM set to those launched at the same locations yields very similar statistics.

### **Deducing energy spectra from drifters**

*Joe LaCasce (University of Oslo)*

Relative dispersion is often quantified using the second order velocity structure function, the mean square velocity difference on pairs of particles. However, the results are frequently ambiguous in terms

of turbulent inertial ranges. We explore why this is so, and demonstrate how pair velocities can instead be used to estimate energy spectra directly.

## Session 6 Abstracts

### **Lagrangian transport in three-dimensional unsteady flows with (perturbed) periodic lines**

*Michel Speetjens (Eindhoven University of Technology)*

Periodic lines and their response to perturbations are key to the Lagrangian dynamics of many three-dimensional (3D) time-periodic flows. However, studies on this subject in general concern highly idealized flows and/or investigate dynamics in terms of a canonical representation based on linearization in the direct proximity of periodic lines. An important question in the light of applications is to what extent such approaches connect with realistic flow situations. This is addressed by investigating (perturbed) periodic lines in experimentally realizable 3D time-periodic flows.

Analyses reveal that each periodic line has a “zone of influence” in which it exclusively governs the dynamics. Here Lagrangian motion is, irrespective of the existence of a global invariant, restricted to local invariant surfaces in which essentially 2D Hamiltonian dynamics occurs. Moreover, the dynamics in (almost) the entire zone of influence admits reconciliation with the beforementioned canonical form through a linearizing transformation. This extends the link between physical and canonical space from the direct proximity of the periodic line to (almost) the entire zone of influence.

The canonical form facilitates systematic exploration of the impact of weak perturbation on the dynamics throughout (almost) the entire zone of influence. Thus various response scenarios and mechanisms can be identified as a function of the nature of the perturbation and the properties of the periodic lines. This encompasses several (in part ill-understood) phenomena studied in literature, demonstrating that the canonical form adequately captures the full richness of the dynamics.

Key findings on (perturbed) periodic lines are supported by laboratory studies on 3D Lagrangian dynamics of tracer particles. This experimentally demonstrates the existence of periodic lines. Moreover, this provides first direct experimental evidence of the existence and universality of a certain response scenario to weak perturbations.

### **Scalar density evolution with Lagrangian measures**

*Wenbo Tang (Arizona State University)*

In integrable shear flows, the second order moments of a passively advected and diffusive scalar can be computed analytically to obtain the spatial- and temporal-dependent effective diffusivity. In non-integrable flows, such analytical results are not accessible. In this talk, we discuss a semi-analytic framework, which reconstructs scalar density evolution from finite-time Lagrangian measures. We pay close attention to the stretching and shearing components of the deformation tensor, both playing important roles in shaping scalar patches. With proper choice of time-scale for mapping the scalar

density forward, we show that the scalar density field can be resolved without having to solve the transport equation.

### **A spectral clustering approach to coherent Lagrangian vortex detection**

*Daniel Karrasch (Technische Universität München)*

One of the ubiquitous features of real-life turbulent flows is the existence and persistence of coherent vortices. Here we show that such coherent vortices can be extracted as clusters of Lagrangian trajectories. To this end, we extract coherent vortices from a graph built from trajectory data, using tools from spectral graph theory. Our method is able to recognize the number of present coherent vortices as well as their location without a priori knowledge. We illustrate the performance of our technique by identifying coherent Lagrangian vortices in several two- and three-dimensional flows. The talk is based on joint work with Alireza Hadjighasem, Hiroshi Teramoto, and George Haller, available under [arxiv.org/abs/1506.02258](https://arxiv.org/abs/1506.02258).

## Session 7 Abstracts

### **Polar rotation angle identifies vortices in unsteady flows**

*Mohammad Farazmand (Massachusetts Institute of Technology)*

We propose rotation inferred from the polar decomposition of the flow gradient as a diagnostic for elliptic (or vortex-type) invariant regions in unsteady flows. We consider here two- and three-dimensional flows, in which polar rotation can be characterized by a single angle. For this polar rotation angle (PRA), we derive explicit formulas using the singular values and vectors of the flow gradient. We find that closed level sets of the PRA reveal vortices in great detail, and singular level sets of the PRA uncover vortex centers.

This is joint work with George Haller.

### **Uncovering Lagrangian vortices objectively from the vorticity**

*George Haller (ETH Zürich)*

Rotationally coherent Lagrangian vortices can be defined as tubes of deforming fluid elements that complete equal bulk material rotation relative to the mean rotation of the fluid domain. Initial positions of such tubes turn out to coincide with tubular level surfaces of the Lagrangian-Averaged Vorticity Deviation (LAVD), the trajectory integral of the normed difference of the vorticity from its spatial mean. LAVD-based vortices are objective and their centers are precisely the observed cyclonic attractors for light inertial particles in oceanic flows. A similar result holds for heavy particles in anticyclonic LAVD vortices. We illustrate the use of the LAVD to detect rotationally coherent Lagrangian vortices objectively in several two- and three-dimensional flows.

## **Objective detection of vortices and their evolution in 2D and 3D flows**

*Melissa Green (Syracuse University)*

We study the formation and shedding of vortices in two vortex-dominated flows in order to detect coherent structures objectively (i.e., in a frame invariant fashion) in massively-separated flow. We employ a recently developed objective definition and extraction technique for rotationally coherent Lagrangian vortices. This method renders material vortex boundaries as outermost convex level surfaces of the Lagrangian-Averaged Vorticity Deviation (LAVD), i.e., the trajectory integral of the normed deviation of the vorticity from its spatial mean. We also employ the derivative of the LAVD, the Instantaneous Vorticity Deviation (IVD), to uncover instantaneous Eulerian vortex boundaries in an objective fashion. These Eulerian vortex boundaries, therefore, remain the same in all possible rotating and translating unsteady frames. The multiple methods we use identify and track both leading edge and trailing edge vortices as they form and shed. This helps to describe the relationship between vortex dynamics and the loss of lift during dynamic stall on a 2D flat plate undergoing a 45 degree pitch-up maneuver, and to describe the dynamic evolution of a hairpin vortex in a three-dimensional channel flow simulation.

## **Session 8 Abstracts**

### **Lagrangian Coherent Structures and Eulerian Eddy Fluxes in the East Pacific**

*Ryan Abernathey (Columbia University)*

Two very different perspectives on the nature of mesoscale (10-300 km) eddies exist in the oceanographic literature. On one hand, ocean modelers emphasize the Eulerian concept of the "eddy flux" (i.e. Reynolds flux) arising from time-varying small-scale correlations between the velocity field and an advected scalar (e.g. the eddy heat flux). This quantity is important because it contributes significantly to the climatological budgets of heat, salt, nutrients, etc. On the other hand, observationalists, and, increasingly, applied mathematicians, conceptualize "eddies" as discrete coherent structures which may trap fluid and propagate spatially. This study attempts to unify these two perspectives by asking, what is the role of Lagrangian coherent structures in the Eulerian eddy flux?

We examine the East Pacific sector, from 180 - 130 E longitude, for which previous studies have examined the Eulerian eddy flux in great detail. We use the AVISO surface geostrophic velocity dataset to drive the advection of a dense array of Lagrangian particles (over 8 million); from these trajectories, Rotationally Coherent Vortices (RCVs) are identified using the objective, frame-invariant definition of Haller et al. (2016). The meridional eddy diffusivity is computed from the full trajectory data using Taylor's formula, and from this, the relative contribution of the RCVs to the net meridional flux is quantified. The coherent vortices are found to make a relatively minor contribution to the net meridional flux, indicating the ocean mesoscale eddy transport is achieved primarily through more disorganized motions. These results contrast with the findings of some previous studies which, using non-objective eddy tracking techniques, may have overestimated the contribution of coherent mesoscale eddies to the Eulerian eddy flux.

## **Dynamics of time-dependent transport boundaries in rotating stratified Euler flow**

*A. D. Kirwan, Jr. (SMSP, University of Delaware)*

The problem considered here is quantification of time-dependent flow boundaries in stratified flow. The velocity field is specified as an analytic solution to the time dependent stratified Euler equations on an f-plane. Results are presented for model parameters scaled for ocean submesoscale dynamics. Particle trajectories live on both cyclonic and anti-cyclonically winding tori. The simplicity of the velocity field is in contrast to the intricate structure of the transport boundary surfaces. They exhibit remarkable depth dependency and vibrate at super-inertial frequencies. Implications for analyses of ocean circulation data are offered.

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## **Session 9 Abstracts**

### **A minimization principle for the description of modes associated with finite-time instabilities**

*Themis Sapsis (Massachusetts Institute of Technology)*

We introduce a minimization formulation for the determination of a finite-dimensional, time-dependent, orthonormal basis that captures directions of the phase space associated with transient instabilities. While these instabilities have finite lifetime, they can play a crucial role either by altering the system dynamics through the activation of other instabilities or by creating sudden nonlinear energy transfers that lead to extreme responses. However, their essentially transient character makes their description a particularly challenging task. We develop a minimization framework that focuses on the optimal approximation of the system dynamics in the neighbourhood of the system state. This minimization formulation results in differential equations that evolve a time-dependent basis so that it optimally approximates the most unstable directions. We demonstrate the capability of the method for two families of problems: (i) linear systems, including the advection–diffusion operator in a strongly non-normal regime as well as the Orr–Sommerfeld/Squire operator, and (ii) nonlinear problems, including a low-dimensional system with transient instabilities and the vertical jet in cross-flow. We demonstrate that the time-dependent subspace captures the strongly transient non-normal energy growth (in the short-time regime), while for longer times the modes capture the expected asymptotic behavior. Joint work with Hessam Babaei.

### **Transport between two fluids across their mutual flow interface: the streakline approach**

*Sanjeeva Balasuriya (University of Adelaide)*

Mixing between initially coherent blobs of two different fluids must be initiated by fluid transporting across the mutual fluid interface. In general, there is no necessity for the physical flow barrier between

the fluids to be associated with extremal or exponential attraction as might be revealed by applying Lagrangian coherent structures, finite-time Lyapunov exponents or other methods on the fluid velocity. Cross-interface transport can be achieved by imposing unsteady velocity agitations in the interface region. It is shown that streaklines are key to understanding the breaking of the interface, and a theory for locating the relevant streaklines is presented. The streaklines are numerically verified in several examples. The relationship to the unsteady advective transport between the two fluids is established.

## **Hyperbolic Neighborhoods in Unsteady Flow**

*Nicholas Ouellette (Stanford University)*

Hyperbolic points and their unsteady generalization (hyperbolic trajectories) drive the exponential stretching that is the hallmark of nonlinear and chaotic flow. In infinite-time flows, the stable and unstable manifolds attached to each hyperbolic trajectory mark fluid elements that asymptote towards the hyperbolic trajectory, and which will therefore eventually experience exponential stretching. In an unsteady finite-time flow, however, hyperbolic trajectories (which move around in the flow) need not remain hyperbolic for all time. We introduce a new way to determine their region of influence, which we term a hyperbolic neighborhood, which marks fluid elements whose dynamics are instantaneously dominated by the hyperbolic trajectory. Fluid elements traversing a flow experience exponential boosts in stretching while within these time-varying regions. We demonstrate our method with several analytical examples, as well as with experimental data from a quasi-two-dimensional laboratory flow.