Introduction to Focus Issue: Objective Detection of Coherent Structures

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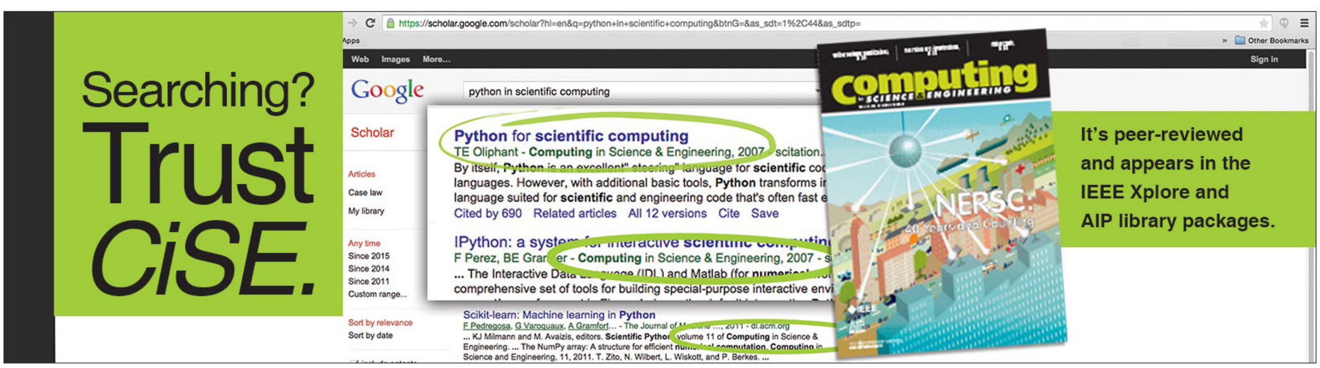

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Introduction to Focus Issue: Objective Detection of Coherent Structures

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Coherent structures are persistent localized features in time-varying fields. Some examples of physical fields in which coherence is of interest include pressure, temperature, and density of a deforming continuum, such as water, air, a granular material or a solid body. A first-order approximation to coherence in any (generally diffusive) physical field is coherence in the deformation of the carrier material. To date, a number of mathematical and ad-hoc methods have been developed, all striving to highlight coherent *material* regions, i.e., temporally coherent sets formed by trajectories of material particles in the phase space. The practitioner, however, is hard pressed to decide which of these methods is suitable for investigating a yet unknown flow. This is a crucial question in real-world applications, such as real-time forecasting and now-casting for environmental decision-making and control.

A simple, yet universal, principle narrows down the panoply of methods significantly to those that are at least consistent with their stated goal of detecting material (or Lagrangian) structures. This principle is objectivity, i.e., invariance under changes of the observer.[10,13,14] This means that a coherent material set identified by an objective criterion in one observer frame should come out to be the same material set when the same criterion is applied in any other observer frame. For instance, sharply visible material coherent structures (fronts) block the transport of algae populations into certain regions of the ocean surface.[20] An observer on a cruising ship or another one on a circling airplane will visually identify the same material points in the front even though those material points will traverse on different paths in the frames of the two observes.

Most coherent structures, however, are not so directly visible as the above example of a front. For instance, boundaries of coherent material eddies embracing and transporting volumes of ocean water with different salinity or temperature are notoriously difficult to identify.[4] The two observers mentioned above could then apply a material coherent structure detection method to the velocity field measured in their own frames. A third observer could also perform the same exercise from the shore (also in a rotating frame, given the motion of the earth). Clearly, if any of these observers reports a different assessment for the coherent material eddy boundary, the coherent structure method all three observers use is not self-consistent and hence is unreliable. More generally, objectivity is a necessary condition for the reliability of any coherent structure method. Any non-objective method, even if it is an exact mathematical criterion, can at best be a sufficient condition, and hence can produce false negatives. Indeed, if the conditions of a non-objective method for coherence are not satisfied in a chosen frame, they may still well be satisfied in another frame. The practitioner would have to check infinitely many different frames to exclude, with certainty, the possibility of a false negative signalled by the method in one frame. A non-objective method without rigorous mathematics is even less helpful: it can produce both false positives and false negatives for material coherent structures.

To be clear, frame-invariance does not mean that each observer should see or measure exactly the same thing. Rather, it means that their conclusions and observations about material deformation should be consistent with each other: they should transform into each other through the same Euclidean coordinate change that connects the observers. By the same token, if a quantity is not tied to material response, there is no ground for demanding it to be objective. For instance, why should velocity or acceleration (notions inherently defined relative to a frame) be frame-invariant? An example of a related common misinterpretation, or misunderstanding, is as follows: "Even Newton's equation of motion and the Navier-Stokes equations are frame-dependent, so why would we need frame-invariance as a requirement for concepts classifying sets of material trajectories arising from these equations?" The answer is that the Newton and Navier-Stokes equations are, in fact, based on objective physical principles for the motion of particles. These governing equations generate physically the same material trajectories, no matter what moving frame is used to evaluate them. The principle of material invariance is actually the one that tells us how to transform the governing equations from one frame to another! Thus, by construction, coordinate representations of the trajectories transform into each other under the frame change. At the same time, specific terms in the Newton and Navier-Stokes equations change in different frames. They change precisely in a way to ensure that the solutions of the equations define physically

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the same material trajectories after the frame change. There is no need for the velocity and acceleration terms in these equations to be individually objective, but there is a clear need for these equations to generate the same material motion in any two frames. Otherwise, different observers would obtain different conclusions about material behavior. This would imply a lack of well-defined physics behind Newton's or the Navier-Stokes equations, given that there is no absolute inertial frame to which we could always resort for the correct result.

The importance of coherent structures extends beyond the already discussed areas of solids and fluids to granular flows and molecular dynamics, and even to dynamical systems describing electrical circuits and financial markets. Granular flows and molecular dynamics also involve material behavior and hence are subjected to the aforementioned requirement of objectivity. Euclidean observer changes and objectivity, however, have little meaning for electric circuits and finance models. That is not to say that anything goes for coherent structure detection in these disciplines. Indeed, research areas using non-material dynamical systems have their own self-consistency requirements to uphold (e.g., conservation laws for circuits, no-arbitrage for finance) that may often be more stringent than objectivity. In this introduction, however, we refrain from discussing self-consistency requirements for coherent structure detection outside areas concerned with material behavior.

Objectivity for coherent structure detection in stochastic material processes deserves a separate mention. Typical outputs of such detection algorithms are sets or scalar fields. In either case, the coherent structure detection methods may have its own internal stochastic component. This component is not visible to an observer, so there is no particular reason for it to be objective, as long as the same sets or scalar fields are detected in all observer frames.

To summarize, we say that an algorithm for coherent material structure detection is objective if the same algorithm, applied in all frames rotating and translating relative to each other, returns the same coherent material structures. That is, the independently detected coherent structures in different frames transform into each other under the same time-dependent rotations and translations as the frames do. This sounds like a trivial requirement, but its use is relatively recent in fluid dynamics,[11,12] perhaps the most important driver of the development of coherent structure methods over the past decade or so. As a result, an alarming number of currently advocated coherent structure detection methods in fluids fail the test of objectivity. Some of these methods claim to give the correct answer to coherent transport and mixing only in the frame of interest but not necessarily in others. These methods invariably fail on appropriate counterexamples, because material transport and mixing, when self-consistently defined and analyzed, must be frame-invariant.

This Focus Issue seeks to aid users and developers of coherence detection methods by surveying the state of the art in objective coherent structure detection, with an emphasis on mathematically well-founded techniques. Several leading researchers in the fields have contributed with results briefly summarized below.

Allshouse and Peacock[1] revisit the scenario of extracting finite-time Lyapunov exponent (FTLE) ridges, providing a robust and practical algorithm for so doing. Good practices for calculation of the FTLE field are also discussed. Once extracted, the challenges of classifying the types of deformation associated with an FTLE ridge are rigorously investigated.[2]

The effect of inertia on the dynamics of inertial particles in rotating two-dimensional flow is investigated by Beron-Vera *et al*.[3] The authors predict attraction or repulsion to and from coherent Lagrangian eddies in the ocean, depending on the rotational sign of eddy and the positive or negative buoyancy of the massive object. They illustrate their predictions on observations of *Sargassum* patterns and float behavior.

Budisic and Thiffeault[5] present a topological descriptor for the finite-time growth of a closed loop in a two-dimensional flow. Braid theory is applied to the materially advected loop to fit an exponential growth rate to the complexity growth of the corresponding braid. The dependence of this growth rate on various computational parameters is explored for Aref's blinking vortex.

Froyland and Junge[7] develop a numerical approach that speeds up the implementation of a transfer operator method designed for detecting coherent sets in purely advective flow. The method minimizes the boundary size of the coherent sets relative to the volume of these sets, and the mesh-free numerical approach is based on radial basis function collocation. This results in accelerated convergence of the transfer operator method, as the authors illustrate on several simple examples. This work complements another article in this issue, by Williams *et al*.[23]

A simple numerical method to identify finite-time coherent regions from incomplete and sparse trajectory data is developed by Froyland and Padberg-Gehle.[8] Their method utilizes an objective spatio-temporal clustering approach to locate trajectories that stay together in approximately spherical regions of the phase space over a finite time interval. Beyond simple examples, the authors also illustrate their results on actual ocean drifter data.

The flow field from simulations of a two-dimensional, incompressible viscous flow of an undulatory, self-propelled swimmer is analyzed for the detection of coherent Lagrangian vortices in the wake by Huhn *et al*.[15] This analysis enables them to dissect the driving momentum transfer mechanisms. At moderate Reynolds number, they find that the resulting flow structures are characterized by unsteady separation and alternating vortices in the wake.[16]

Karrasch[17] shows that in two-dimensional incompressible flows attracting Lagrangian coherent structures (LCSs) can appear as ridges of the forward FTLE field. This raises the issue of characterization of attracting LCSs from a forward finite-time Lyapunov analysis. Recent results by Haller and Sapsis are extended, regarding the relation between forward and backward maximal and minimal FTLEs, to both the full finite-time Lyapunov spectrum and to stretch directions.

Mahoney and Mitchell[18] extend the recent variational principle of shearless LCS from passive advection to finite-time reactive flows. Their main result is an objective criterion for burning LCS (bLCS) which form one-way barriers to reaction front propagation. The new theory is tested on a time-independent, double-vortex channel with an opposing draft.

A study of material separation along attracting LCSs emanating from the vicinity of a no-slip boundary in an unsteady fluid flow is the focus of the article by Miron *et al*.[19] Previous results have only treated the case when the LCS originates from the wall itself. This study considers off-wall separation and locates the actual separation point with a combination of objective analytic and diagnostic methods.

The geometry of Lagrangian motion and material barriers in a time-dependent, three-dimensional, Ekman-driven, rotating cylinder flow, which serves as an idealization for an isolated oceanic eddy and other overturning cells with cylindrical geometry in the ocean and atmosphere, is studied by Rypina *et al*.[21] The Lagrangian geometry changes near the resonant tori of the unforced flow, whose frequencies are rationally related to the forcing frequencies. Multi-scale analytical expansions are used to simplify the flow in the vicinity of resonant trajectories and to investigate the resonant flow geometries.

A study of an atmospheric blocking event over a finite time duration is presented by Ser-Giacoma *et al*.[22] They discretise a two-dimensional phase space into boxes and construct a transition matrix between boxes using Ulam's method. From this Markov chain model, they compute paths that are most probable over a specified time interval. The method is applied to 12 h of atmospheric data over Northern Europe around the heatwave of July 2010.

Solomon and Gowen[9] present experimental studies of reaction front propagation in a single vortex flow with an imposed external wind. Reaction fronts triggered in or in front of the moving vortex form persistent structures that are seen experimentally for time-independent (constant motion), time-periodic, and time-aperiodic flows. These results are examined with the use of burning invariant manifolds that act as one-way barriers to front motion in the flows. The authors also explore the usefulness of finite-time Lyapunov exponent fields as an instrument for analyzing front propagation behavior in a fluid flow.

Williams *et al*.[23] describe a probabilistic approach to determining finite-time coherent sets using a mesh-free basis. Their constructions may be viewed as a collocation-based approximation via thin-plate splines of the transfer operator method of Froyland,[6] which used Galerkin projection onto piecewise constant basis functions. The aim of Williams *et al*. is to work with sparser data, and examples are presented for the double-gyre, the Bickley jet, and flow in the Sulu Sea. This work complements another article in this issue, by Froyland and Junge.[7]

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